Research Summary

Libin Liang　Zhejiang University
E-mail:3110000271@zju.edu.cn　Phone: 9198025084

Advisor: Tao Pang　Associate Professor
Department of Mathematics, North Carolina State University

Title: Mortgage Backed Securities (MBS) Evaluations and Risk Analysis

Abstract: Mortgage-backed securities (MBS), once a legend of financial instruments, occupied lots of attention and also held one of the fastest growing, as well as one of the largest financial markets in the United States before financial crisis in 2008. However, since it triggered the financial crisis in 2008, people are more aware of the significance of MBS's risk. In this research, we will investigate the risk associated with different MBS products and their pricing methods.

The key to price the MBS is to evaluate the prepayment risk in the future. In our research, first, to get the real prepayment behavior in the market, we collect mortgage data from the Fannie Mae data base and generate the conditional prepayment rate (CPR) and the cash flow with the data, regarding it as a real MBS product. Then we test different models of interest rate and then price the MBS product by utilizing Monte-Carlo Simulation. Further, we will test some prepayment interest models and calibrate the models with real data. Meanwhile, we also build up a new prepayment model based on the real data. Last, we will analyze the risks asscoiated with MBS products based on the option adjusted spread (OAS) and some characteristics such as effective duration, effective convexity and simulated average life. Through our research, we expect to create our own prepayment model and price certain MBS bonds by simulating that model.

Research goal:

(1).To price MBS mortgages and to create our own prepayment model based on real data.

(2)To do the sensitivity analysis with the results and to calculate effective duration and effective convexity.

(3)To conduct option adjusted spread (OAS) analysis in order to compare bonds with different structures.

Research method:

(1).Collect and Analyze real data

Real data collected from the Fannie Mae data base, including current interest rate, loan age, remaining months to legal maturity, adjusted remaining months to legal maturity and so on. Then we cleaned the data and derived the cash flow , the balance and the CPR from the data.

(2).Generate the interest path with interest model

We utilized Courtadon model to generate interest rate paths. Courtadon model is a

stochastic differential eaquation indicate the relation between interest rate and time. Then we use numeric method to simulate the path of the interest rate.
The general form of the interest model can be described as follow:

$$dr_t = \kappa(\theta - r_t)dt + \sigma r^{\alpha} dB_t$$

$$r(0) = r_0$$

In our calculation, the interest rate model is Courtadon's model which can be expressed as:

$$dr_t = \kappa(\theta - r_t)dt + \sigma r dB_t$$

$$r(0) = r_0$$

The discrete form of the formula is:

$$\Delta r_t = \kappa(\theta - r_t)\Delta t + \sigma r_t B\sqrt{\Delta t}$$

$$r(0) = r_0$$

(3).Build the new prepayment model by modulating the old prepayment model in the reference paper.

(4). Simulate the situation in the mortgage market with prepayment model
With the generated interest rate and the prepayment model, we can calculate the prepayment rate and get the estimated cash flow in the future. With the estimated cash flow of the product, we can get one estimated price of the product with the discounted factor.

(5).Use Monte-Carlo simulation to get the price of the MBS or CMO product
As the method above, we can get one estimated price of the MBS or CMO product. To get a more accurate price, we do the simulation thousands of times and calculate the average price as the price of the product.

(6).Sensitivity analysis
To measures factors such as the securities credit risk and liquidity, we use OAS analysis. We change different CPR and get the OAS by Monte-Carlo simulation. To measures the price sensitivity of a bond to a small change in interest rate, we calculate the effective duration and effective convexity. With the OAS, effective duration and effective convexity, we can know how the prepayment risk distribute in the CMO structure.
OAS analysis: OAS analysis is designed to measure the yield spread of a fixed income security that is not attributed to imbedded options. For a given price, the OAS is the spread such that the average of the distribution of the values equals the price. The bigger the OAS is, the bigger the prepayment risk is.

Effective duration: The effective duration measures the price sensitivity of a bond to a small change in interest rates. The formula of the effective duration:

$$\text{Effective duration} = \frac{100}{P_0} * \frac{P_- - P_+}{2 * \Delta y}$$

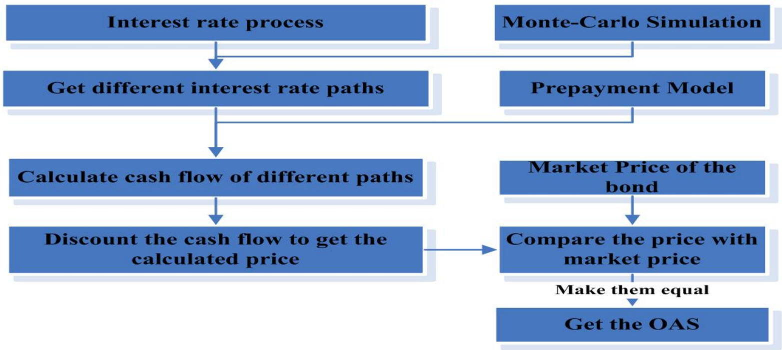
The bigger the effective duration, the more sensitive the bond is to a change in interest rate.

Effective convexity: The effective convexity describes the rate at which duration changes in response to changes in interest rates. The formula of the effective convexity:

$$\text{Convexity}=\frac{100}{P_0}*\frac{P_+ + P_- - 2*P_0}{(\Delta y)^2}$$

The bigger the effective convexity is, the more sensitive the effective duration is in response to changes in interest rates.

The process of the simulation and the analysis can be described in the figure below:



Results and Discussion

1.Results of Collecting and Analyzing real data

By dealing with the data, we get the balance, the cash flow and the CPR in the real market. The graphs below show the results.

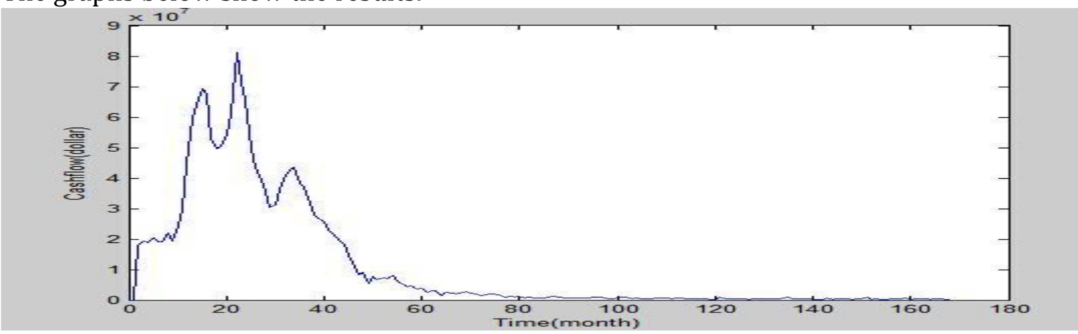


Figure1(Cash flow)

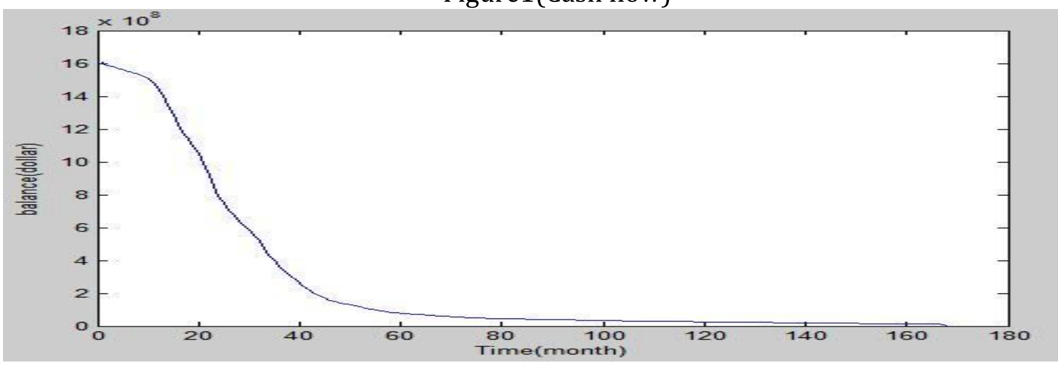


Figure2(balance)

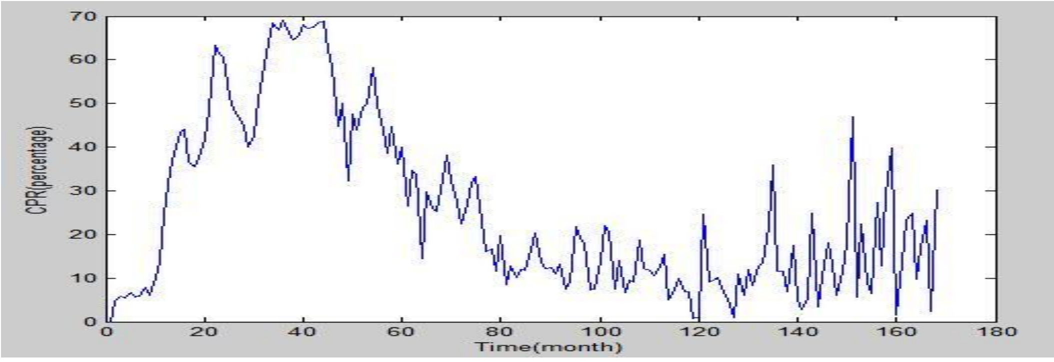


Figure3(CPR)

From the figure1, we found that the cash flow is significant huge between 10th to 50th month. In that period, the balance is not so small and the CPR is so high. This two factor can explain the huge cash flow. In the period after the 80th month, the balance is small and the CPR is also small. This can be explained by the burnout effect. When the balance is small, the money can be saved by prepayment behavior is small so the prepayment rate is low.

2.The interest path generated by the interest model

By utilizing the Courtadon's model and the given parameter, one path of market interest rate can be generated.

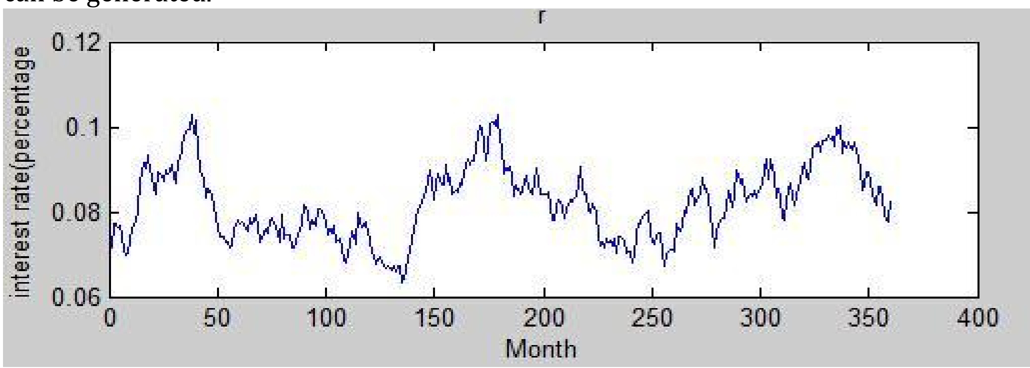


Figure4(Interest rate)

Model: $dr_t = \kappa(\theta - r_t)dt + \sigma r dB_t$

$r(0) = r_0$

Parameter: $\kappa = 0.29368, \sigma = 0.11, \theta = 0.08, r_0 = 0.0715$

As can be seen in the graph, the path always back to the long-run average 8%(θ).

3.New prepayment model

Refinancing incentive

Four factors are included in our prepayment model. They are refinancing incentive, seasoning, month of the year and premium burnout. The prepayment rate can be expressed as the multiplicative formula below:

$$CPR = (refinancing - incentive) * (Age - Multiplier) * (Month - Multiplier) * (Burnout - Multiplier)$$

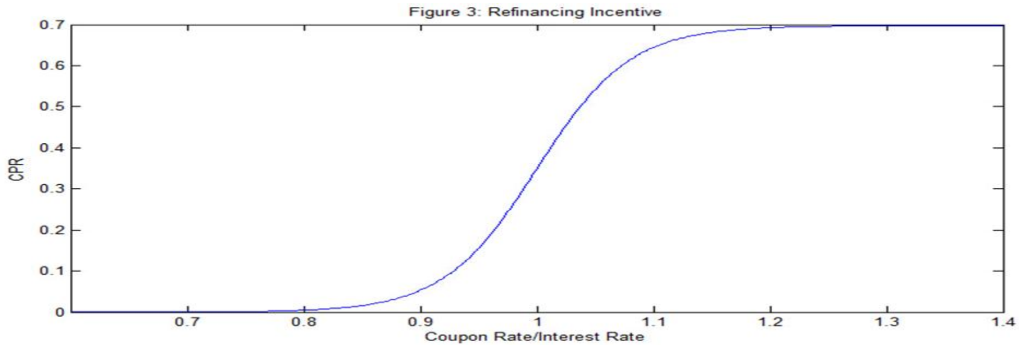


Figure5(Refinancing Incentive-C/R))

Refinance incentive is a factor that determined by the interst rate.According to reference, the normal shape of C/R-CPR graph is similar to Logistic function .So we use logistic function to math the curve. We get the parameters from the real data.

Monthly multiplier

Monthly multiplier is used to take into consideration of the time of the year. Borrowers have different prepayment behavior in different time of the year.

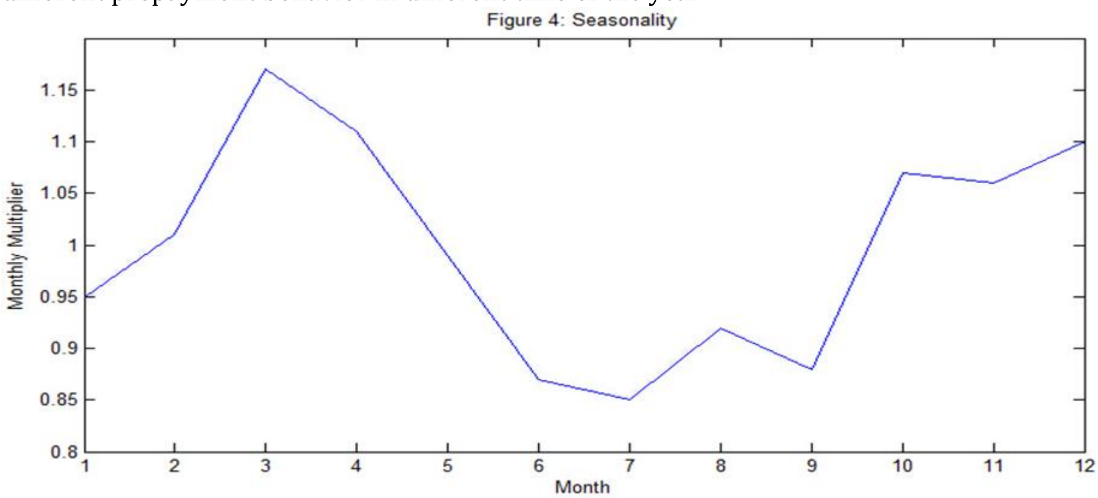


Figure6(Monthly multiplier)

The monthly multipliers of our prepayment model are shown above. They are derived by the data from Fannie Mae data base.

Premium Burnout

When the balance is getting less and less, the prepayment rate will be diminish. So we use a exponential function to describe this trend.

$$BM = 0.7 \times e^{(1-B_0/B_t)} + 0.3$$

The graph of the burnout factor is shown below:

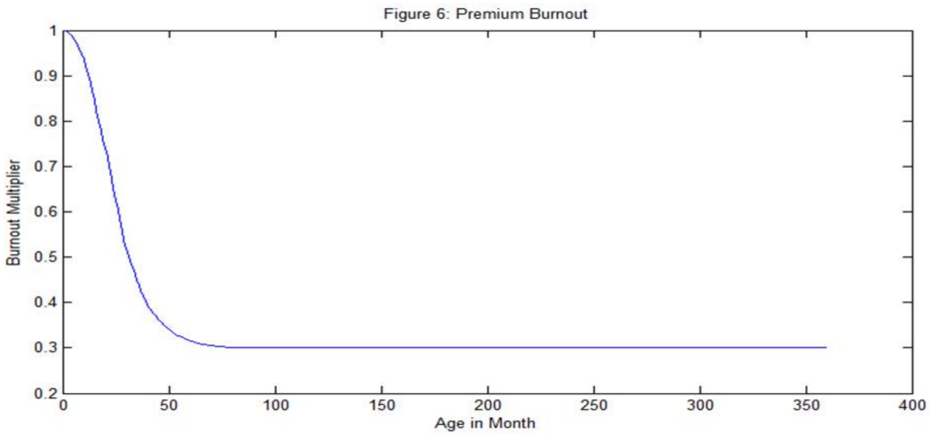


Figure7(Burnout)

Seasoning

The newer loans tend to prepay slower. To describe this factor, we use the function

$$Age(t) = \min(\frac{t}{30}, 1)$$

The graph of the seasoning factor is shown below:

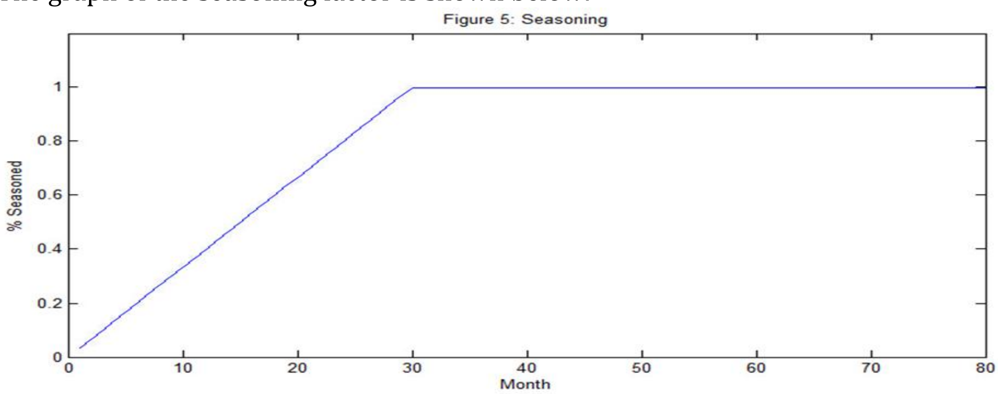


Figure8(Seasoning)

4.Result of the simulation

With the prepayment model we built and the chosen interest rate model, the CPR of the interest rate path above can be shown in the graph below(apply to a MBS product with 1000000 dollars as an initial balance):

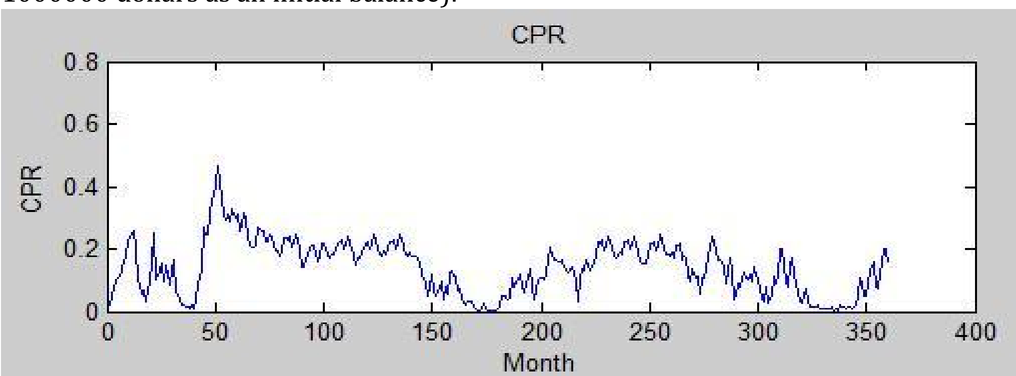


Figure9(CPR generated by simulation)

As can be seen in the graph, when the interest rate is high the CPR is low. The period between 45th to 55th month can be a cogent example. This was resulted from the refinancing incentive.

And we can also generate the cash flow of the market.

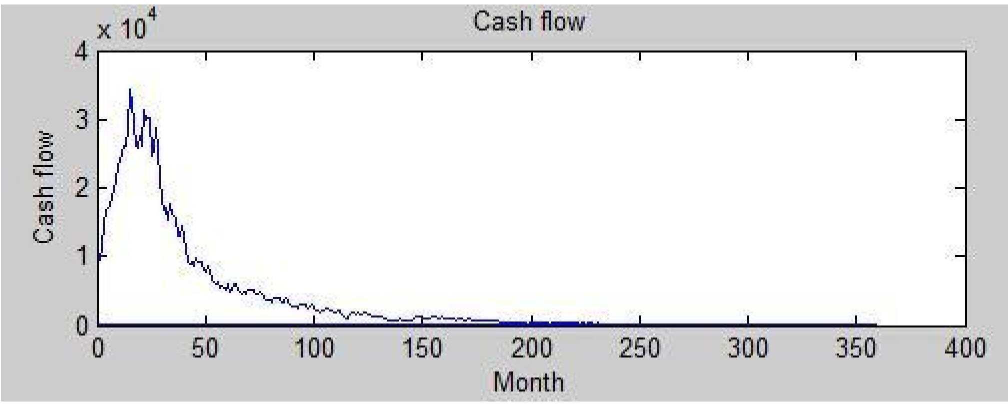


Figure10(Cash flow generated by simulation)

By comparing figure1 and figure6 we can find the shape of the graph of the cash flow generated by simulation is quiet similar to the one of the cash flow generated by the real market.

5.Price the CMO product and analyze the risk by simulation

The CMO product we used in the simulation can be shown in the table below.

Collateral	1000000	8.50%
Bond A	200000	8.50%
Bond B	300000	8.00%
Bond C	350000	8.20%
Bond D	150000	7.80%

The cash flow of the SEQ product can be shown below:

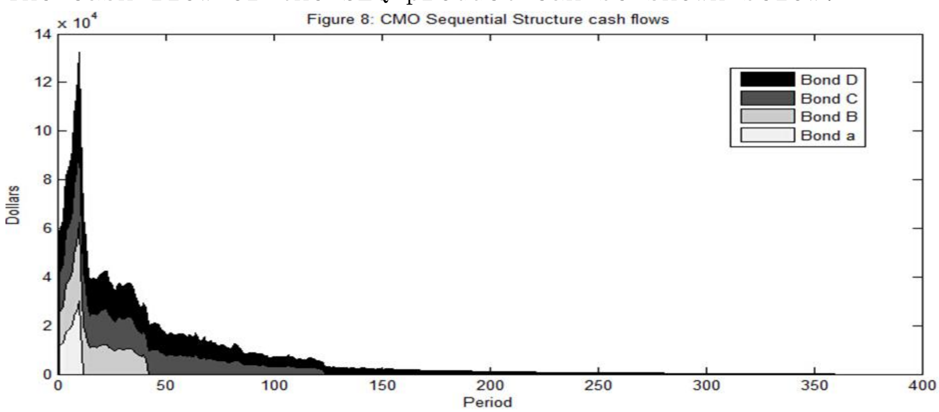


Figure11(Cash flow of the CMO product)

The price we get by simulation and the sensitivity analysis of the SEQ product(one class of the CMO product) was shown in the graph below:

Prepayment at 100% of Prepayment Model					
	Market Price	OAS (in bps)	Simulated Average Life	Effective Duration	Effective Convexity
Collateral	$1,012,005.03	0	4.40890	3.61830	-0.25812
Class					
A	$201,917.33	0	0.83070	0.88543	-0.06008
B	$302,704.07	0	1.98600	2.34196	-0.26528
C	$356,966.95	0	4.69360	4.23964	-0.35547
D	$150,416.68	0	13.36150	8.38026	-0.27679

As the graph shows above, the structure of the CMO product is an important factor to the price sensitivity of a bond to a small change in interest rates. The earlier the bond receives principle payment, the less the effective duration is. It means that the bond is less sensitive to the change in interest rate if the bond receives principle payment earlier.

The results when PSA change:

Prepayments at 80% and 120% of Prepayment Model				
	New OAS		Simulated Average Life	
	80%	120%	80%	120%
Collateral	1.231	-1.255	5.2377	3.8607
Class				
A	1.649	-1.685	0.9331	0.7567
B	1.570	-1.430	2.2894	1.7958
C	0.619	-0.934	5.6691	4.0870
D	0.590	-0.751	15.8670	11.6010
	Effective Duration		Effective Convexity	
	80%	120%	80%	120%
Collateral	4.0021	3.6162	-0.2511	-0.2636
Class				
A	0.9975	0.5986	-0.0695	-0.0533
B	2.6465	2.1657	-0.3028	-0.2455
C	4.7498	3.8908	-0.3522	-0.3682
D	8.9923	7.9679	-0.1357	-0.2922

The earlier the bond receives principle payment, the bigger the OAS absolute value is. It means that the earlier the bond receives principle payment, the more significant the risk of the bond is. The bond A and bond B have the similar prepayment risk while bond B and bond C have smaller risk. From this analysis, we can change the structure CMO product to create bonds with different prepayment risk. We can redirect the cash flow or we can change the amount of the bond. From the graph show above, we can also find that the smaller the PSA, the longer the average life of the bond is. And we also can conclude that the higher the prepayment rate, the more sensitive the bond is to the change in interest rate.

When we try more different PSA, we find that the linear relation between OAS of bond A and PSA.

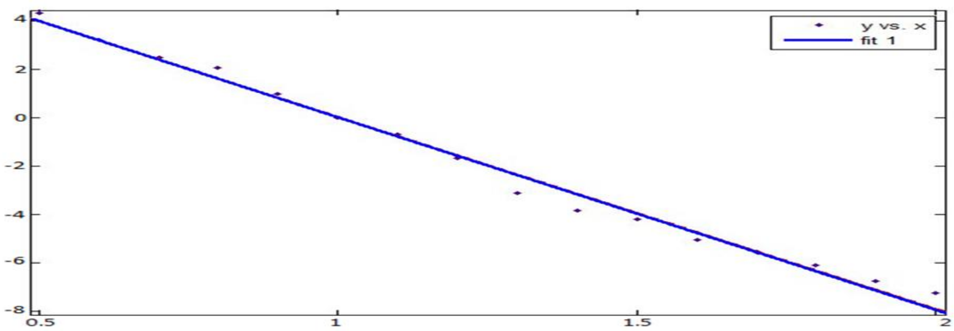


Figure12(OAS-PSA of bond A)

The regression equation is: $OAS = -7.935 \times CPR + 7.965$

It means that we can find the approximate OAS of the bond with the equation and it will be more efficient.